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Precision Flavour Physics with $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$

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Abstract

We show that a combined analysis of $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$ allows for new physics tests practically free of form factor uncertainties. Residual theory errors are at the level of several percent. Our study underlines the excellent motivation for measuring these modes at a Super Flavour Factory.

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1 Introduction

The standard model of quark flavour physics has successfully passed all experimental tests to date. This includes the observation of a substantial number of rare processes and CP asymmetries, which are consistently accounted for within the Cabibbo-Kobayashi-Maskawa (CKM) description of quark mixing. On the other hand, many essential features of the standard model, most notably in the flavour sector, are still not satisfactorily understood on a more fundamental level. Deviations from standard expectations, which could guide us towards a better understanding, appear to be small in general in view of the basic agreement between theory and observations. In this situation precision tests of flavour physics become increasingly important, which motivates current efforts to build a Super Flavour Factory [1,2]. Such a facility will enable an exciting program in B physics [3,4].

One of the best opportunities in this respect could be provided by the study of $b \to s\nu\bar{\nu}$ transitions, induced by interactions at very short distances. Theoretically ideal would be an inclusive measurement of $B \to X_s\nu\bar{\nu}$, where the hadronic matrix element can be accurately computed using the heavy-quark expansion. Unfortunately, because of the missing neutrinos, an inclusive experimental determination of the decay rate is probably unfeasible. More promising is the measurement of exclusive channels such as $B \to K\nu\bar{\nu}$, $B \to K^*\nu\bar{\nu}$. In this case a clean theoretical interpretation requires, however, the control of nonperturbative hadronic form factors. Direct calculations of form factors suffer from sizable uncertainties. Additional experimental input to eliminate nonperturbative quantities can therefore be very useful. Important examples are the related decays $K \to \pi\nu\bar{\nu}$, where the hadronic matrix element can be eliminated with the help of $K^+ \to \pi^0 e^+\nu$ using isospin symmetry. As discussed in [5], a similar role could be played by the semileptonic mode $B \to \pi e\nu$ for the rare decay $B \to K\nu\bar{\nu}$. This strategy is limited by the breaking of $SU(3)$ flavour symmetry of the strong interaction, which is also difficult to estimate with high accuracy.

In this paper we propose to perform a combined analysis of the rare decays $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$. As we shall discuss, this option has several advantages for controlling hadronic uncertainties. It allows us to construct precision observables for testing the standard model and for investigating new physics effects. In particular neither isospin nor $SU(3)$ flavour symmetry are required and form factor uncertainties can be eliminated to a large extent.

The paper is organized as follows. Section 2 summarizes the experimental status. Section 3 collects basic theoretical results. It includes a discussion of $B \to K$ form factors, weak annihilation and nonperturbative corrections in $B \to Kl^+l^-$, and the background for $B^- \to K^-\nu\bar{\nu}$ from $B^- \to \tau^-\bar{\nu}_\tau \to K^-\nu_\tau\bar{\nu}_\tau$. Precision observables are discussed in section 4. Section 5 comments on the effects of new physics and conclusions are presented in section 6. Further details on form factors relations and on weak annihilation are collected in the appendix.

Table 1: The normalized q^2-spectrum for $B \to K l^+ l^-$. Shown are the partial branching fractions in six bins of q^2 (or $s = q^2/m_B^2$) from [10], normalized by the central value of the integrated branching fraction in (3). These quantities are denoted by $\Delta B/B$ in the table.

$q^2[\mathrm{GeV}^2]$	s	$\Delta B/B$
0.00–2.00	0.00–0.07	0.169 ± 0.038
2.00–4.30	0.07–0.15	0.096 ± 0.027
4.30–8.68	0.15–0.31	0.208 ± 0.042
10.09–12.86	0.36–0.46	0.115 ± 0.031
14.18–16.00	0.51–0.57	0.079 ± 0.033
> 16.00	> 0.57	0.204 ± 0.042

2 Experimental status

In this section we summarize briefly the current experimental situation. For the branching ratios of the neutrino modes $\bar{B} \to \bar{K}\nu\bar{\nu}$ only upper limits are available at present. They read [6,7,8,9]

$$B(B^- \to K^-\nu\bar{\nu}) < 14 \cdot 10^{-6} \tag{1}$$

$$B(\bar{B}^0 \to \bar{K}^0\nu\bar{\nu}) < 160 \cdot 10^{-6} \tag{2}$$

Here CP averaged branching fractions are understood. We note that the limit is more stringent for the B^- channel.

The most accurate experimental results for $B \to K l^+ l^-$ are from Belle [10]. The extrapolated, non-resonant branching fraction is measured to be

$$B(B \to K l^+ l^-) = (0.48^{+0.05}_{-0.04} \pm 0.03) \cdot 10^{-6} \tag{3}$$

consistent with results from BaBar [11]. The recent paper [10] also contains information on the q^2-spectrum in terms of partial branching fractions for six separate bins. The results for the normalized q^2-spectrum, adapted from [10], are given in Table 1.

3 Theory of $\bar{B} \to \bar{K}\nu\bar{\nu}$ and $\bar{B} \to \bar{K} l^+ l^-$

3.1 Dilepton-mass spectra and short-distance coefficients

We define the kinematic quantities

$$s = \frac{q^2}{m_B^2} \qquad r_K = \frac{m_K^2}{m_B^2} \tag{4}$$

where q^2 is the dilepton invariant mass squared and m_B is the mass of the B meson. The kinematical range of q^2 and its relation with the kaon energy E_K are given by

$$4m_l^2 \simeq 0 \leq q^2 \leq (m_B - m_K)^2 \qquad\qquad q^2 = m_B^2 + m_K^2 - 2m_B E_K \tag{5}$$

We also use the phase-space function

$$\lambda_K(s) = 1 + r_K^2 + s^2 - 2r_K - 2s - 2r_K s \tag{6}$$

The differential branching fractions for $\bar{B} \to \bar{K}\nu\bar{\nu}$ and $\bar{B} \to \bar{K}l^+l^-$ can then be written as follows:

$$\frac{dB(\bar{B} \to \bar{K}\nu\bar{\nu})}{ds} = \tau_B \frac{G_F^2 \alpha^2 m_B^5}{256\pi^5} |V_{ts}V_{tb}|^2 \, \lambda_K^{3/2}(s) f_+^2(s) \, |a(K\nu\nu)|^2 \tag{7}$$

$$\frac{dB(\bar{B} \to \bar{K}l^+l^-)}{ds} = \tau_B \frac{G_F^2 \alpha^2 m_B^5}{1536\pi^5} |V_{ts}V_{tb}|^2 \, \lambda_K^{3/2}(s) f_+^2(s) \left(|a_9(Kll)|^2 + |a_{10}(Kll)|^2\right) \tag{8}$$

Here τ_B is the B-meson lifetime, G_F the Fermi constant, $\alpha = 1/129$ the electromagnetic coupling and V_{ts}, V_{tb} are elements of the CKM matrix. A second contribution to the amplitudes proportional to $V_{us}^* V_{ub}$ has been neglected. It is below 2% for $\bar{B} \to \bar{K}l^+l^-$ and much smaller still for $\bar{B} \to \bar{K}\nu\bar{\nu}$.

The factorization coefficient $a(K\nu\nu)$ is simply given by a short-distance Wilson coefficient at the weak scale, C_L^ν, [5]

$$a(K\nu\nu) = C_L^\nu = -\frac{1}{\sin^2 \Theta_W} \eta_X X_0(x_t) \tag{9}$$

where X_0 is an Inami-Lim function [12] and $x_t = m_t^2/M_W^2$, with $m_t = \bar{m}_t(m_t)$ the $\overline{\mathrm{MS}}$ mass of the top quark. The factor $\eta_X = 0.994$ accounts for the effect of $\mathcal{O}(\alpha_s)$ corrections [13]. At this order the residual QCD uncertainty is at the level of 1-2% and thus practically negligible.

The factorization coefficient $a_9(Kll)$ contains the Wilson coefficient $\tilde{C}_9(\mu)$ combined with the short-distance kernels of the $\bar{B} \to \bar{K}l^+l^-$ matrix elements of four-quark operators evaluated at $\mu = \mathcal{O}(m_b)$. The coefficient $a_9(Kll)$ multiplies the local operator $(\bar{s}b)_{V-A}(\bar{l}l)_V$. At next-to-leading order (NLO) the result can be extracted from the expressions for the inclusive decay $\bar{B} \to X_s l^+l^-$ given in [12,14,15], where also the Wilson coefficients and operators of the effective Hamiltonian and further details can be found. The NLO coefficient reads

$$\begin{aligned} a_9(Kll) = {} & \tilde{C}_9 + h(z, \hat{s}) \, (C_1 + 3C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ & -\frac{1}{2} h(1, \hat{s}) \, (4C_3 + 4C_4 + 3C_5 + C_6) \\ & -\frac{1}{2} h(0, \hat{s}) \, (C_3 + 3C_4) + \frac{2}{9} \, (3C_3 + C_4 + 3C_5 + C_6) + \frac{2m_b}{m_B} C_7 \end{aligned} \tag{10}$$

Here

$$\tilde{C}_9(\mu) = P_0 + \frac{Y_0(x_t)}{\sin^2\Theta_W} - 4Z_0(x_t) + P_E E_0(x_t) \tag{11}$$

is the Wilson coefficient in the NDR scheme, P_0, P_E are QCD factors and E_0, Y_0, Z_0 are Inami-Lim functions. The function $h(z, \hat{s})$, $z = m_c/m_b$, $\hat{s} = q^2/m_b^2$ arises from one-loop electromagnetic penguin diagrams, which determine the matrix elements of four-quark operators. In contrast to $\tilde{C}_9$ the quantity $a_9(Kll)$ is scale and scheme independent at NLO. To this order the coefficients C_i, $i = 1, \ldots 7$ in (10) are needed only in leading logarithmic approximation (LO). Note that here the labeling of C_1 and C_2 is interchanged with respect to the convention of [12].

The coefficient $a_{10}(Kll)$ is

$$a_{10}(Kll) = \tilde{C}_{10} = -\frac{1}{\sin^2\Theta_W} Y_0(x_t) \tag{12}$$

3.2 Form factors

The long-distance hadronic dynamics of $\bar{B} \to \bar{K}\nu\bar{\nu}$ and $\bar{B} \to \bar{K}l^+l^-$ is contained in the matrix elements

$$\langle \bar{K}(p')|\bar{s}\gamma^\mu b|\bar{B}(p)\rangle = f_+(s)\,(p+p')^\mu + [f_0(s) - f_+(s)]\,\frac{m_B^2 - m_K^2}{q^2} q^\mu \tag{13}$$

$$\langle \bar{K}(p')|\bar{s}\sigma^{\mu\nu} b|\bar{B}(p)\rangle = i\frac{f_T(s)}{m_B + m_K}\,[(p+p')^\mu q^\nu - q^\mu (p+p')^\nu] \tag{14}$$

which are parametrized by the form factors f_+, f_0 and f_T. Here $q = p - p'$ and $s = q^2/m_B^2$. The term proportional to q^μ in (13), and hence f_0, drops out when the small lepton masses are neglected as has been done in (7) and (8). The ratio f_T/f_+ is independent of unknown hadronic quantities in the small-s region due to the relations between form factors that hold in the limit of large kaon energy [16,17]

$$\frac{f_T(s)}{f_+(s)} = \frac{m_B + m_K}{m_B} + \mathcal{O}(\alpha_s, \Lambda/m_b) \tag{15}$$

Here we have kept the kinematical dependence on m_K in the asymptotic result. In contrast to f_+ the form factor f_T is scale and scheme dependent. This dependence is of order α_s and has been neglected in (15). Within the approximation we are using we may take $\mu = m_b$ to be the nominal scale of f_T.

We remark that the same result for f_T/f_+ is also obtained in the opposite limit where the final state kaon is soft, that is in the region of large $s = \mathcal{O}(1)$. This follows from the asymptotic expressions for f_+ and f_T in heavy hadron chiral perturbation theory [18,19,20,21,22]. From this observation we expect (15) to be a reasonable approximation in the entire physical domain. This is indeed borne out by a detailed analysis of QCD sum rules on the light cone [23], which cover a range in s from 0 to 0.5. Relation (15) is further discussed in appendix A.

The ratio f_T/f_+ enters (10) as a prefactor of C_7 from the matrix element of the corresponding magnetic-moment type operator Q_7 [5,12]. In writing (10) the relation (15) has already been used to eliminate f_T/f_+. Since the C_7 term contributes only about 13% to the amplitude $a_9(Kll)$, the impact of corrections to (15) will be greatly reduced. A 15% uncertainty, which may be expected for the approximate result (15), will only imply an uncertainty of 2% for $a_9(Kll)$ or the $\bar{B} \to \bar{K}l^+l^-$ differential rate. In practice, this leaves us with the form factor $f_+(s)$ as the essential hadronic quantity for both $\bar{B} \to \bar{K}\nu\bar{\nu}$ and $\bar{B} \to \bar{K}l^+l^-$.

The main emphasis of the present study is on the construction of clean observables, which are, as far as possible, independent of hadronic input. We will therefore consider suitable ratios of branching fractions where the form factor $f_+(s)$ is eliminated to a large extent. In order to assess the residual form factor uncertainties in these cases, but also to estimate absolute branching fractions, it will be useful to have an explicit parametrization of the form factor at hand. We employ the parametrization proposed by Becirevic and Kaidalov [27] in the form

$$f_+(s) \equiv f_+(0)\,\frac{1-(b_0+b_1-a_0b_0)s}{(1-b_0s)(1-b_1s)} = f_+(0)[1+a_0b_0\,s+\mathcal{O}(s^2)] \tag{16}$$

The parameter b_0 is given by

$$b_0 = \frac{m_B^2}{m_{B_s^*}^2} \approx 0.95 \qquad \text{for} \quad m_{B_s^*} = 5.41\,\text{GeV} \tag{17}$$

b_0 represents the position of the B_s^* pole and will be treated as fixed, following [27]. The remaining three parameters a_0, b_1 and $f_+(0)$ have been determined from QCD sum rules on the light cone (LCSR) in [23]

$$f_+(0) = 0.304 \pm 0.042, \qquad a_0 \approx 1.5, \qquad b_1 = b_0 \tag{18}$$

We will treat all three as variable parameters. This also includes b_1, slightly generalizing the expressions from [23] where b_1 is fixed at b_0. The value for $f_+(0)$ in (18) is obtained from the relation [23]

$$f_+(0) = 0.331 \pm 0.041 + 0.25(\alpha_1(1\,\text{GeV}) - 0.17) \tag{19}$$

using the updated value [24,25] for the Gegenbauer coefficient $\alpha_1(1\,\text{GeV}) = 0.06 \pm 0.03$ as quoted in [26].

With b_0 fixed, the parameter a_0 introduced in (16) determines the slope of the form factor at small s. We remark that the LCSR method is appropriate for the low-s region, which will be of particular interest for us. For completeness we give the relation of our parameters $f_+(0)$, a_0, b_1 to the original parameters c_B, $\alpha \equiv \alpha_B$, $\gamma \equiv \gamma_B$ from [27]:

$$f_+(0) = c_B(1-\alpha_B)\,, \qquad a_0 = \frac{\gamma_B - \alpha_B}{\gamma_B(1-\alpha_B)}\,, \qquad \frac{b_0}{b_1} = \gamma_B \tag{20}$$

As discussed in [27], the large-energy limit for the kaon implies the relation $\gamma_B = 1/\alpha_B$ or, equivalently, $a_0 b_0 = b_0 + b_1$.

We determine next our default ranges for the shape parameters a_0 and b_1, which will be employed in the subsequent phenomenological analysis. Three main pieces of information will be used: The experimental data on the q^2 spectrum in Table 1, the LCSR results in (18), and asymptotic results for the form factor at maximum s,

$$s_m = \left(1 - \frac{m_K}{m_B}\right)^2 \tag{21}$$

The third constraint will lead to a relation between a_0 and b_1. It follows from the asymptotic expression for $f_+(s_m)$

$$f_+(s_m) = \frac{g f_B m_B}{2 f_K (m_K + \Delta)} \qquad\qquad \Delta = m_{B_s^*} - m_B \tag{22}$$

which can be derived within heavy-hadron chiral perturbation theory [18,19,20,21,22]. The largest uncertainty in (22) is due to the BB_s^*K coupling g, sometimes also normalized as $g_{BB_s^*K} = 2m_B g/f_K$, which is not known precisely. For the analogous, $SU(3)$ related quantity $g_{BB^*\pi}$ a range of $g_{BB^*\pi} = 42 \pm 16$ is quoted in [27]. This corresponds roughly to $g = 0.6 \pm 0.2$. In view of this uncertainty, and since the main purpose here is the estimate of typical numbers, we have neglected subleading corrections to (22), which may be sizeable [20]. We recall that g is of order unity in the large-m_B limit.

Equating (22) with $f_+(s_m)$ from (16) and using

$$1 - b_0 s_m = \frac{2(m_K + \Delta)}{m_B} \tag{23}$$

we obtain

$$\frac{1 - s_m(b_0 + b_1 - a_0 b_0)}{1 - s_m b_1} = \frac{g f_B}{f_+(0) f_K} \equiv c_0 \approx 2.5 \pm 1.0 \tag{24}$$

We note that the denominator of the first term in (24), $1 - s_m b_1$, scales as $1/m_B$ in the heavy-quark limit, whereas the numerator remains of order unity (if a_0 is not too far below its typical value of 1.5). The first term then scales as m_B, consistent with the heavy-quark scaling of the second expression.

The constraint (24) can also be put in the form

$$c_0 - 1 = \frac{a_0 - 1}{\frac{1}{s_m b_0} - \frac{b_1}{b_0}} \tag{25}$$

Numerically we have $1/(s_m b_0) = 1.279$. Within the uncertainty of c_0, displayed in (24) above, (25) implies a correlation between the shape parameters a_0 and b_1/b_0.

Independently of such theory constraints we might ultimately want to extract the form factor shape from experimental data. In this spirit, we have investigated how well different values of $(a_0, b_1/b_0)$ fit the current Belle measurements of the dilepton-mass spectrum in $B \to K l^+ l^-$. For this purpose we show in Table 2 the χ^2-function

Table 2: Values of χ^2 for various combinations of the form factor shape parameters a_0 and b_1, determined from a comparison with the Belle data on the q^2 spectrum of $B \to K l^+ l^-$ (see Table 1).

b_1/b_0 \ a_0	1.0	1.2	1.4	1.6	1.8	2.0
0.5	20.2	14.7	11.0	8.8	7.5	7.1
0.6	20.2	14.2	10.3	8.0	6.9	6.7
0.7	20.2	13.5	9.4	7.1	6.2	6.4
0.8	20.2	12.7	8.3	6.2	5.7	6.4
0.9	20.2	11.8	7.1	5.3	5.5	7.0
1.0	20.2	10.5	5.7	4.7	6.2	9.2

$$\chi^2(a_0, b_1) = \sum_{i=1}^{6} \frac{(y_i - F_i(a_0, b_1))^2}{\sigma_i^2} \tag{26}$$

where the y_i, $i = 1, \ldots, 6$, are the experimental values for the normalized, partial branching fractions $\Delta B/B$ in each of the six bins, the σ_i are the corresponding errors, and the F_i are the theoretical expressions depending on (a_0, b_1). It is clear that the experimental data are at present not accurate enough to allow for a precise determination of the form factor shape. However, the situation should improve in the future. At the moment our analysis merely serves to illustrate the general method. Nevertheless, some regions of parameter space are already disfavoured, in particular low values of a_0, which, as discussed above, is consistent with theoretical expectations. We also observe that the best fit is obtained for $a_0 = 1.6$, $b_1/b_0 = 1$. A comparison of the best-fit shape of the theoretical spectrum with the Belle data is shown in Fig. 1

Combining the information above, we adopt the following default ranges for the shape parameters

$$1.4 \leq a_0 \leq 1.8 \qquad 0.5 \leq b_1/b_0 \leq 1.0 \tag{27}$$

with

$$a_0 = 1.6 \qquad b_1/b_0 = 1.0 \tag{28}$$

as our reference values. Within the range (27) for a_0 and b_1/b_0 the parameter c_0 in (25) takes values between 1.5 and 3.9, compatible with (24). Our default parameters (28) are also consistent with the LCSR results [23] quoted in (18).

3.3 $\bar{B} \to \bar{K} l^+ l^-$: Weak annihilation

As pointed out in [28], the exclusive decay $\bar{B} \to \bar{K} l^+ l^-$ receives contributions from weak annihilation diagrams already at leading order in the heavy-quark limit. In spite of this

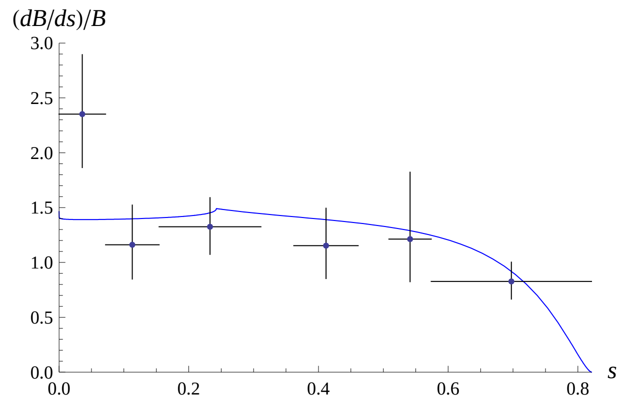

Figure 1: The shape of the $\bar{B} \to \bar{K} l^+ l^-$ spectrum, $(dB/ds)/B$, from the Belle data summarized in Table 1 (crosses) and from theory with the best-fit shape parameters $a_0 = 1.6$, $b_1/b_0 = 1$ (solid curve).

their impact is numerically small because of a strong CKM suppression (for the charged mode) or small Wilson coefficients (for the neutral mode). In this section we will quantify the size of weak annihilation. Since this contribution is small we work to leading order in α_s.

First we consider the case of $\bar{B}^0 \to \bar{K}^0 l^+ l^-$, and the region of low $s \sim \Lambda/m_b$. Weak annihilation can then only come from penguin operators giving rise to the transition $b\bar{d} \to s\bar{d}$. A virtual photon emitted from one of the quarks produces the lepton pair. The leading annihilation contribution in the heavy-quark limit is generated by the gluon-penguin operators Q_3 and Q_4 [12]. This effect can be evaluated using the methods of QCD factorization [28,29] as discussed in more detail in appendix B. The resulting correction to the coefficient $a_9(Kll)$ reads

$$\Delta a_{9,WA,34} = \left(C_4 + \frac{1}{3} C_3\right) \frac{8\pi^2 Q_d f_B f_K}{m_B f_+(s)} \int_0^\infty d\omega \, \frac{\phi_-(\omega)}{\omega - s\, m_B - i\epsilon} \tag{29}$$

Here f_B, f_K are the meson decay constants and $Q_d = -1/3$ is the charge of the down-quark in the initial state. The leading light-cone distribution amplitudes of the B meson can be expressed by two functions, $\phi_\pm(\omega)$, of which only ϕ_- enters the integral in (29). For annihilation contributions scale dependent quantities such as the Wilson coefficients $C_{3,4}$ will be evaluated at a hard-collinear scale of $\mu_h = \sqrt{\mu \Lambda_h}$, where $\mu = \mathcal{O}(m_b)$ and $\Lambda_h = 0.5\,\text{GeV}$, following [29].

Our result (29) for weak annihilation agrees with eq. (68) of [28], once it is adapted to the case of a pseudoscalar K meson as described immediately after eq. (69) of [28].[1]

[1] A factor of $(-2m_b/M_B)$ is missing in front of the annihilation term on the r.h.s. of eq. (41) in [28]. We thank Thorsten Feldmann for confirmation.

For estimates of the annihilation effect we employ the model functions [30]

$$\phi_+(\omega) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0} \qquad\qquad \phi_-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0} \tag{30}$$

where $\omega_0 = \mathcal{O}(\Lambda_{\rm QCD})$ serves to parametrize the uncertainty related to $\phi_\pm$. A summary of general properties of these wave functions can be found in [28,17]. They are satisfied by the parametrizations in (30). With $\phi_-(\omega)$ from (30), and denoting by Ei(z) the exponential integral function, the integral in (29) can be expressed as [28]

$$\lambda_{B,-}^{-1}(s) \equiv \int_0^\infty d\omega\, \frac{\phi_-(\omega)}{\omega - s\, m_B - i\epsilon} = \frac{1}{\omega_0}\, e^{-s m_B/\omega_0}\, [-{\rm Ei}(s m_B/\omega_0) + i\pi] \tag{31}$$

At a typical value of $s = 0.1$ we have $a_9 = 3.96 + 0.05i$ and $\Delta a_{9,WA,34} = -0.036 + 0.034i$. The correction (29) is seen to reduce the real part of a_9 by a small amount, which leads to a corresponding reduction of the branching fraction. This holds if $s \geq 0.37\omega_0/m_B \approx 0.025$. Since the imaginary part of a_9 is much smaller than the real part, its impact on the decay rate is entirely negligible. Practically it is thus of no consequence that the imaginary part of the correction is comparable to the one of a_9 and that ${\rm Im}\, a_9$ is rather uncertain. Of particular interest for us is the size of the annihilation effect on the partially integrated branching fraction. For $\omega_0 = (0.350 \pm 0.150)\,{\rm GeV}$, $m_b/2 \leq \mu \leq 2m_b$ the reduction of the branching fraction integrated within $0.03 \leq s \leq 0.25$ is at most 1%, which is indeed very small.

Weak annihilation contributions to $\bar{B}^0 \to \bar{K}^0 l^+ l^-$ also come from the remaining two QCD penguin operators Q_5 and Q_6. Because these have a chiral structure different from Q_3, Q_4, their contribution to weak annihilation is formally suppressed in Λ/m_b. It turns out, however, that the suppression is not very effective numerically in this particular case. A similar situation is familiar from the factorizing matrix elements of Q_5 and Q_6 for B decays into a pair of light pseudoscalar mesons [29]. The explicit calculation of the annihilation correction to $a_9(Kll)$ from Q_5 and Q_6 proceeds as before and yields

$$\Delta a_{9,WA,56} = -\left(C_6 + \frac{1}{3}C_5\right) \frac{16\pi^2 Q_d f_B f_K \mu_K}{m_B^2 f_+(s)} \int_0^\infty d\omega\, \frac{\phi_-(\omega)}{\omega - s\, m_B - i\epsilon} \tag{32}$$

In comparison to (29) the correction in (32) carries, apart from the Wilson coefficients, a relative factor of $-2\mu_K/m_B \approx -0.75$, where $\mu_K = \mu_\pi = m_\pi^2/(m_u + m_d)$ [29]. Here m_u, m_d are the $\overline{\rm MS}$ masses evaluated at the scale μ_h. As anticipated, this relative factor is not small, although it is of $\mathcal{O}(\Lambda/m_b)$. A typical value for (32), at $s = 0.1$, is $\Delta a_{9,WA,56} = 0.045 - 0.043i$. The sign of the real part is opposite to the case of $\Delta a_{9,WA,34}$ such that there is a tendency of the two contributions to cancel. Because $|C_6 + C_5/3|$ is larger than $|C_4 + C_3/3|$, it is possible that (32) even dominates over (29). Of course, (32) is formally a power correction and other power corrections to weak annihilation do exist. However, the numerically large factor $2\mu_K/m_B$ is special to the annihilation matrix element of $Q_6 = -2(\bar{d}b)_{S-P}(\bar{s}d)_{S+P}$ with the presence of (pseudo)scalar currents. Taking into account the single power correction (32) thus appears justified. Adding

both corrections, (29) and (32), the net effect on the branching ratio integrated from $s = 0.03$ to 0.25 is a tiny enhancement. This enhancement remains below about 0.3% for $\omega_0 = (0.350 \pm 0.150)\,\mathrm{GeV}$ and $m_b/2 \le \mu \le 2m_b$.

To summarize, the weak annihilation contributions to $\bar{B}^0 \to \bar{K}^0 l^+ l^-$, which arise from QCD penguin operators, are negligibly small in practice, even though they are a leading-power effect. If the presumably dominant (chirally enhanced) power correction (32) is also included, the total impact of weak annihilation is further reduced.

In the case of the charged mode $B^- \to K^- l^+ l^-$ the weak annihilation terms from QCD penguin operators are given by the expressions (29) and (32) with the replacement of the quark charge $Q_d \to Q_u$. Numerically the effect then receives an additional factor of -2, which still yields a negligible correction at the level of about one percent. Weak annihilation through the tree operators Q_1 and Q_2, which exists only for the charged mode, comes with large Wilson coefficients, but also with a strong Cabibbo suppression. This correction reads

$$\Delta a_{9,WA,12u} = -\frac{V^*_{us}V_{ub}}{V^*_{ts}V_{tb}} \left(C_1 + \frac{1}{3}C_2\right) \frac{8\pi^2 Q_u f_B f_K}{m_B f_+(s)} \int_0^\infty d\omega \, \frac{\phi_-(\omega)}{\omega - s\, m_B - i\epsilon} \tag{33}$$

With (33) the branching ratio integrated from $s = 0.03$ to 0.25 is reduced by less than about 0.6% for $\omega_0 = (0.350 \pm 0.150)\,\mathrm{GeV}$ and $m_b/2 \le \mu \le 2m_b$. Taking the three contributions from $Q_{1,2}$, $Q_{3,4}$ and $Q_{5,6}$ together, the reduction remains below 1%. Within an uncertainty of this order, weak annihilation is therefore negligible for $B^- \to K^- l^+ l^-$ as well.

3.4 $\bar{B} \to \bar{K} l^+ l^-$: Nonperturbative corrections

In this section we comment on the theoretical framework for $\bar{B} \to \bar{K} l^+ l^-$ and on non-perturbative effects beyond those that are contained in the form factors.

It is well known that, because of huge backgrounds from $\bar{B} \to \bar{K}\psi^{(\prime)} \to \bar{K} l^+ l^-$, the region of q^2 containing the two narrow charmonium states $\psi = \psi(1S)$ and $\psi' = \psi(2S)$ has to be removed by experimental cuts from the q^2 spectrum of $\bar{B} \to \bar{K} l^+ l^-$. The overwhelming background from ψ and ψ' is related to a drastic failure of quark-hadron duality in the narrow-resonance region for the *square* of the charm-loop amplitude, as has been discussed in [31]. Nevertheless, the parts of the q^2 spectrum below and above the narrow-resonance region remain under theoretical control and are sensitive to the flavour physics at short distances. A key observation here is that the amplitude is largely dominated by the semileptonic operators

$$\begin{aligned} Q_9 &= (\bar{s}b)_{V-A}(\bar{l}l)_V \\ Q_{10} &= (\bar{s}b)_{V-A}(\bar{l}l)_A \end{aligned} \tag{34}$$

which have large coefficients $\tilde{C}_9$ and $\tilde{C}_{10}$. These contributions are perturbatively calculable up to the long-distance physics contained in the form factor $f_+(s)$. The $\bar{B} \to \bar{K} l^+ l^-$ matrix elements of four-quark operators, such as $(\bar{s}b)_{V-A}(\bar{c}c)_{V-A}$, are more complicated,

but still systematically calculable. Schematically, the $\bar{B} \to \bar{K} l^+ l^-$ rate is proportional to

$$|\tilde{C}_9 + \Delta_{4q}|^2 + |\tilde{C}_{10}|^2 \tag{35}$$

where Δ_{4q} represents contributions from four-quark operators, for instance charm loops or annihilation effects. In the present discussion we ignore the small contribution from C_7, which has already been discussed in section 3.2. Typical values are $\tilde{C}_9 = 4.2$, $\tilde{C}_{10} = -4.2$ and, for the charm-loop amplitude $\Delta_{4q} \approx (C_1 + 3C_2) h(z, \hat{s}) \approx 0.3$. The last figure corresponds to an average within $0 < s < 0.25$. For large s the charm loop develops an imaginary part, but the magnitude is of similar size. Annihilation effects are negligible as shown in section 3.3. Thus Δ_{4q} is only about a 10% effect, both as a correction to the $\tilde{C}_9$ amplitude and to the total rate. Because this term is numerically subleading the impact of any uncertainties in its evaluation will be suppressed. We briefly discuss the theory of Δ_{4q} in the regions of low and high q^2.

In the *low-q^2 region* Δ_{4q} can be computed using QCD factorization [28]. This approach, which is based on the heavy-quark limit and the large energy of the recoiling kaon, should work well for the real part of the amplitude in view of the experience from two-body hadronic B decays [32] and $B \to K^* \gamma$ [33]. Power corrections of order $\Lambda/m_b \sim 0.1$ in Δ_{4q} give only percent level corrections for the differential rate. The charm loops receive also corrections of order Λ^2/m_c^2 [34], which have been estimated at the level of a few percent for the exclusive decay $B \to K^* \gamma$ [35]. Since the charm loops are relatively less important in $\bar{B} \to \bar{K} l^+ l^-$ by about a factor of five in the rate, the impact of the correction is reduced. On the other hand, the effect increases somewhat as q^2 approaches the resonance region. In the inclusive case $b \to s l^+ l^-$ it amounts to a few percent [36], averaged over the low-q^2 region. We therefore conclude that the Λ^2/m_c^2 correction is unlikely to affect $\bar{B} \to \bar{K} l^+ l^-$ in an appreciable way.

Any quark-level calculation of physical amplitudes involves the concept of quark-hadron duality. There are no indications that this assumption, applied to the charm-loops for small q^2 up to about $7\,\mathrm{GeV}^2$ ($s = 0.25$), would introduce an error in excess of power corrections or perturbative uncertainties [31].

Light-quark loops are generally suppressed by small Wilson coefficients (QCD penguins) or small CKM factors. Violations of local quark-hadron duality could come from the presence of light vector resonances at low q^2. To get an order-of-magnitude estimate we consider the branching ratio of the decay chain $B^- \to K^- \rho^0 \to K^- e^+ e^-$, which is measured to be [6]

$$\begin{aligned} &B(B^- \to K^- \rho^0) \times B(\rho^0 \to e^+ e^-) = \\ &(4.2 \pm 0.5) \cdot 10^{-6} \times (4.71 \pm 0.05) \cdot 10^{-5} = (2.0 \pm 0.2) \cdot 10^{-10} \end{aligned} \tag{36}$$

This contribution arises from the $|\Delta_{4q}|^2$ term in (35) for which quark-hadron duality cannot be expected to hold [31]. However, like similar contributions with other light vector resonances, it clearly gives a negligible contribution. Resonance effects could be more important in the interference of Δ_{4q} with $\tilde{C}_9$ in (35). In this case they are part of the hadronic amplitude that is dual to light-quark loops in the partonic calculation.

Oscillations in s due to light resonances are bound to be small because of the smallness of the light-quark loops. Integration over the low-q^2 region will further reduce such violations of local duality to a negligible level.

In the *high-q^2 region* the appropriate theoretical framework for the computation of Δ_{4q} is an operator product expansion exploiting the presence of the large scale $q^2 \sim m_b^2$. This concept has been used in [22] in analyzing the endpoint region of $b \to sl^+l^-$, which is governed by few-body exclusive modes. A systematic treatment, including the discussion of subleading corrections, has been given in [37]. Power corrections are generally smaller than for low q^2. Terms of order Λ/m_b arise at order α_s [37] and the analogue of the Λ^2/m_c^2 corrections at small q^2 now contribute only at order Λ^2/m_b^2 [36]. More important are perturbative corrections to the leading-power term, which however can be systematically improved. Finally, uncertainties could come from violations of local quark-hadron duality. By duality violation we mean deviations of the OPE calculation, at fixed q^2 and in principle including all perturbative and power corrections, from the real-world hadronic result. Such violations are related in particular to oscillations of Δ_{4q} in s due to higher charmonium resonances. These oscillations are absent in the smooth OPE result. To first order in Δ_{4q} only its real part contributes to (35). Implementing the higher charmonium resonances, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$, and the hadronic $c\bar{c}$ continuum in the approximation of Krüger and Sehgal [38], we estimate the relative amplitude of oscillations in $\tilde{C}_9 + \mathrm{Re}\,\Delta_{4q}$ to be of order 10 to 20%. We expect these local variations to be averaged out when the spectrum is integrated over s [22] such that the residual uncertainty will be somewhat reduced. The s-integration, which is also phenomenologically motivated, effectively produces a smearing that leads to a more 'globally' defined quantity where duality is better fulfilled. As discussed in [31], global duality in this sense cannot be expected to hold for the second order term $|\Delta_{4q}|^2$ in (35). On the other hand, this contribution is numerically very small, at the level of few percent, and duality violations will only have a minor effect. To illustrate this point we consider the decay chain $B^- \to K^-\psi(3770) \to K^-e^+e^-$, which can be viewed as a resonance contribution to $|\Delta_{4q}|^2$. In the case of the narrow charmonium states a similar contribution leads to the very large resonance background mentioned above. Here we have [6]

$$\begin{aligned} &B(B^- \to K^-\psi(3770)) \times B(\psi(3770) \to e^+e^-) = \\ &(4.9 \pm 1.3) \cdot 10^{-4} \times (9.7 \pm 0.7) \cdot 10^{-6} = (4.8 \pm 1.3) \cdot 10^{-9} \end{aligned} \tag{37}$$

This indicates that resonance contributions are rather small, in agreement with our previous remarks. In conclusion, we have argued that duality violations from the resonance region at high q^2 are at a moderate level and should not spoil a precision of theoretical predictions for (partially) integrated branching ratios of $\bar{B} \to \bar{K}l^+l^-$ at the level of several percent. A more detailed investigation of this issue would be of interest and will be given elsewhere.

In the present analysis we ignore higher order electroweak and QED radiative corrections. The latter could modify the decay modes and their ratios presumably at the

M_W [GeV]	$\bar{m}_t(\bar{m}_t)$ [GeV]	m_B [GeV]	m_K [GeV]	$m_{B_s^*}$ [GeV]
80.4	166	5.28	0.496	5.41
$\bar{m}_b$ [GeV]	$\bar{m}_c$ [GeV]	α	$\sin^2\theta_W$	$\lvert V_{ts}^* V_{tb}\rvert$
4.2	1.3	1/129	0.23	0.039
$f_+(0)$	f_B [GeV]	f_K [GeV]	$\Lambda_{\overline{\rm MS},5}$ [GeV]	τ_{B^+} (τ_{B^0}) [ps]
0.304 ± 0.042	0.2	0.16	0.225	1.64 (1.53)

Table 3: Input parameters.

level of several percent. The leading effects could be taken into account if it should be required by the experimental precision.

3.5 $B^- \to K^-\nu\bar{\nu}$: Background from $B^- \to \tau^-\bar{\nu}_\tau \to K^-\nu_\tau\bar{\nu}_\tau$

The decay $B^- \to \tau^-\bar{\nu}_\tau$ followed by $\tau^- \to K^-\nu_\tau$ produces a background for the short-distance reaction $B^- \to K^-\nu\bar{\nu}$, which has been discussed recently in [39]. The branching fractions of $B^- \to \tau^-\bar{\nu}_\tau$ and $\tau^- \to K^-\nu_\tau$ are given by

$$\begin{aligned} B(B^- \to \tau^-\bar{\nu}_\tau) &= \tau_B \frac{G_F^2 m_B m_\tau^2 f_B^2}{8\pi}|V_{ub}|^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \\ &= 0.87\cdot 10^{-4}\left(\frac{f_B}{0.2\,\text{GeV}}\right)^2\left(\frac{|V_{ub}|}{0.0035}\right)^2 \end{aligned} \qquad (38)$$

$$B(\tau^- \to K^-\nu_\tau) = \tau_\tau \frac{G_F^2 m_\tau^3 f_K^2}{16\pi}|V_{us}|^2\left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 = 7.46\cdot 10^{-3} \qquad (39)$$

The numerical values are based on the input parameters in Table 3. The background from the decay chain $B^- \to \tau^-\bar{\nu}_\tau \to K^-\nu_\tau\bar{\nu}_\tau$ gives a contribution to the dilepton-mass spectrum, which can be written as

$$\frac{dB(B^- \to K^-\nu_\tau\bar{\nu}_\tau)_{bkgr}}{ds} = B(B^- \to \tau^-\bar{\nu}_\tau)\, B(\tau^- \to K^-\nu_\tau)\,\frac{2t((1-t)(t-r_K)-ts)}{(1-t)^2(t-r_K)^2} \qquad (40)$$

where we used (4) and $t \equiv m_\tau^2/m_B^2$. The spectrum in (40) extends from $s = 0$ to $s = (1-t)(1-r_K/t) = 0.818$. The maximum s happens to coincide almost exactly with the endpoint of the spectrum in the short-distance decay $B^- \to K^-\nu\bar{\nu}$, $s_m = 0.821$ [39]. Integrated over the full range in s, the phase-space factor in (40) gives 1. This reproduces the result for $B(B^- \to K^-\nu_\tau\bar{\nu}_\tau)_{bkgr}$ in the narrow-width approximation for the intermediate τ lepton.

If the decay sequence $B^- \to \tau^-\bar{\nu}_\tau \to K^-\nu_\tau\bar{\nu}_\tau$ cannot be distinguished experimentally from the short-distance decay $B^- \to K^-\nu\bar{\nu}$, this background should be subtracted from

the measured rate of $B^- \to K^- +$ "invisible" to obtain the true short-distance branching fraction. In the standard model we find from (38) and (39)

$$B(B^- \to K^- \nu_\tau \bar{\nu}_\tau)_{bkgr} = (0.65 \pm 0.16) \cdot 10^{-6} \tag{41}$$

assuming an uncertainty of 25% due to f_B and $|V_{ub}|$. The central value amounts to about 15% of the short-distance branching fraction (44). A subtraction of (41) from the measured branching ratio would then lead to an uncertainty of about $0.16/4.4 = 4\%$ on $B(B^- \to K^- \nu\bar{\nu})$. This error might be further reduced in the future with improved determinations of f_B and $|V_{ub}|$.

Probably the best method to control the background is to use the experimental measurement of $B(B^- \to \tau^- \bar{\nu}_\tau)$. In this way, any new physics component in the latter decay will automatically be removed from the measurement of $B^- \to K^- \nu\bar{\nu}$. This will simplify the new physics interpretation of the measured $B(B^- \to K^- \nu\bar{\nu})$. The present experimental value for $B(B^- \to \tau^- \bar{\nu}_\tau)$ is [7,40,41]

$$B(B^- \to \tau^- \bar{\nu}_\tau)_{exp} = (1.43 \pm 0.37) \cdot 10^{-4} \tag{42}$$

Together with (39) this gives

$$B(B^- \to K^- \nu_\tau \bar{\nu}_\tau)_{bkgr} = (1.1 \pm 0.28) \cdot 10^{-6} \tag{43}$$

The 26% uncertainty in (42) thus implies an error of about 6% in $B(B^- \to K^- \nu\bar{\nu})$, assuming the central value of (44). However, by the time when $B(B^- \to K^- \nu\bar{\nu})$ will be measured at a Super Flavour Factory, $B(B^- \to \tau^- \bar{\nu}_\tau)$ will be simultaneously known with high precision. According to [3,4] the expected accuracy is about 4% or better. Assuming the central values of (42) and (44) as before, the background subtraction will then lead to an error of only about 1% in $B(B^- \to K^- \nu\bar{\nu})$.

We conclude that the background from $B^- \to \tau^- \bar{\nu}_\tau \to K^- \nu_\tau \bar{\nu}_\tau$ pointed out in [39] has to be taken into account for a precise measurement of the short-distance branching fraction $B(B^- \to K^- \nu\bar{\nu})$. It needs to be subtracted from the experimental signal, but this should ultimately be possible with essentially negligible uncertainty. The background discussed here is absent in the case of the neutral mode $\bar{B}^0 \to \bar{K}^0 \nu\bar{\nu}$.

4 Precision observables

4.1 Theory expectations for branching fractions

The input parameters we will use in the present analysis are collected in Table 3.

To begin our discussion of numerial results we consider first the integrated branching ratios of $B^- \to K^- \nu\bar{\nu}$ and $B^- \to K^- l^+ l^-$. For the neutrino mode we find

$$B(B^- \to K^- \nu\bar{\nu}) \cdot 10^6 = 4.4\,^{+1.3}_{-1.1}\,(f_+(0))\,^{+0.8}_{-0.7}\,(a_0)\,^{+0.0}_{-0.7}\,(b_1) \tag{44}$$

We have displayed the sensitivity to the form factor parameters, which are by far the dominant sources of uncertainty. The form factor normalization $f_+(0)$ has the largest impact, while the shape parameters are relatively less important.

The fully integrated, non-resonant $\bar{B} \to \bar{K} l^+ l^-$ branching fraction can be evaluated in a similar way. This quantity corresponds essentially to the experimental result in (3), which has been obtained by cutting out the large background from the two narrow charmonium resonances and by extrapolating the measurements to the entire q^2 range to recover the total non-resonant rate. The precise correspondence between theoretical and experimental results will depend on the details of the cuts and the extrapolation procedure. We will treat the resonance region more carefully later when we study precision observables. For our present discussion we simply identify the integral over the non-resonant spectrum in (8) with the measurement in (3). This appears justified as the error from this identification is expected to be below the experimental uncertainty. Adopting these considerations we compute

$$B(B^- \to K^- l^+ l^-) \cdot 10^6 = 0.58 \,{}^{+0.17}_{-0.15}\,(f_+(0))\,{}^{+0.10}_{-0.09}\,(a_0)\,{}^{+0.00}_{-0.09}\,(b_1)\,{}^{+0.04}_{-0.03}\,(\mu) \tag{45}$$

In addition to the still dominant dependence on the form factor we have in this case a non-negligible perturbative uncertainty, which we estimate in the standard way through a variation of the scale μ between $m_b/2$ and $2m_b$ around the reference value of $\mu = m_b$. The scale dependence is at a rather moderate level of $\pm 6\%$ with NLO accuracy, much smaller than the error from the hadronic parameters. Within sizeable, mainly theoretical uncertainties, the prediction (45) is in agreement with the measurement in (3).

Whereas the individual branching fractions (44) and (45) suffer from large hadronic uncertainties, we expect their ratio to be under much better theoretical control. It is obvious that the form factor normalization $f_+(0)$ cancels in this ratio. Moreover, as illustrated in Fig. 2, the shape of the q^2 spectrum is almost identical for the two modes. This is because the additional q^2-dependence from charm loops in $B \to K l^+ l^-$, compared to $B \to K \nu\bar{\nu}$, is numerically only a small effect outside the region of the narrow charmonium states. As a consequence, also the dependence on the form factor shape will be greatly reduced in the ratio

$$R = \frac{B(B^- \to K^- \nu\bar{\nu})}{B(B^- \to K^- l^+ l^-)} \tag{46}$$

Numerically we find

$$R = 7.59 \,{}^{+0.01}_{-0.01}\,(a_0)\,{}^{+0.00}_{-0.02}\,(b_1)\,{}^{-0.48}_{+0.41}\,(\mu) \tag{47}$$

This prediction is independent of form factor uncertainties for all practical purposes. It is limited essentially by the perturbative uncertainty at NLO of $\pm 6\%$. Using the experimental result in (3), the theory prediction (47), and assuming the validity of the standard model, we obtain

$$B(B^- \to K^- \nu\bar{\nu}) = R \cdot B(B^- \to K^- l^+ l^-)_{exp} = (3.64 \pm 0.47) \cdot 10^{-6} \tag{48}$$

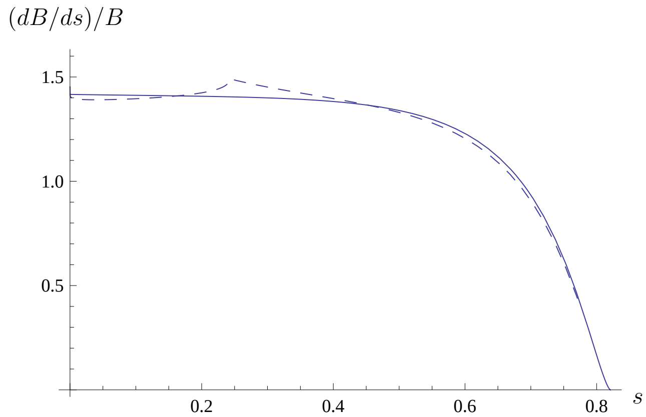

Figure 2: The dilepton invariant-mass spectra for $\bar{B} \to \bar{K}\nu\bar{\nu}$ (solid) and $\bar{B} \to \bar{K}l^+l^-$ (dashed). For easier comparison of the shapes the plotted differential branching fractions, dB/ds versus $s = q^2/m_B^2$, were normalized by their integral B. The reference values have been used for all parameters.

With an accuracy of $\pm 13\%$, limited at present by the experimental error, this result is currently the most precise estimate of $B(B^- \to K^-\nu\bar{\nu})$.

Since isospin breaking effects in the decay rates are very small, the branching ratios for the decays $\bar{B}^0 \to \bar{K}^0\nu\bar{\nu}$ and $\bar{B}^0 \to \bar{K}^0 l^+l^-$ are given by the branching ratios for the corresponding B^- modes multiplied by a factor of $\tau(\bar{B}^0)/\tau(B^-) = 0.93$.

4.2 Precision observables: ratios of branching fractions

In order to obtain theoretically clean observables, the region of the two narrow charmonium resonances $\psi(1S)$ and $\psi(2S)$ has to be removed from the q^2 spectrum of $B \to Kl^+l^-$. This leaves two regions of interest, the low-s region below the resonances, and the high-s region above. For the present analysis we define these ranges as

$$\begin{aligned} &\text{low } s: && 0 \le s \le 0.25 \\ &\text{high } s: && 0.6 \le s \le s_m \end{aligned} \tag{49}$$

The resonance region $0.25 < s < 0.6$ corresponds to the q^2 range $7\,\text{GeV}^2 < q^2 < 16.7\,\text{GeV}^2$. For our standard parameter set the total rate for $B \to K\nu\bar{\nu}$ or $B \to Kl^+l^-$ (non-resonant) is divided among the three regions, low-s, narrow-resonance, high-s, as $35 : 48 : 17$.

We first concentrate on the low-s region, where $B^- \to K^-l^+l^-$ can be reliably calculated. To ensure an optimal cancellation of the form factor dependence, one may restrict also the neutrino mode to the same range in s and define

$$R_{25} \equiv \frac{\int_0^{0.25} ds\; dB(B^- \to K^-\nu\bar{\nu})/ds}{\int_0^{0.25} ds\; dB(B^- \to K^-l^+l^-)/ds} \tag{50}$$

This ratio is determined by theory to very high precision. Displaying the sensitivity to the shape parameters and the renormalization scale one finds

$$R_{25} = 7.60\,{}^{-0.00}_{+0.00}\,(a_0)\,{}^{-0.00}_{+0.00}\,(b_1)\,{}^{-0.43}_{+0.36}\,(\mu) \tag{51}$$

The form factor dependence is seen to cancel almost perfectly in R_{25}. The shape parameters affect this quantity at a level of only 0.5 per mille. One is therefore left with the perturbative uncertainty, estimated here at about $\pm 5\%$ at NLO.

The independence of any form factor uncertainties in R_{25} comes at the price of using only 35% of the full $B^- \to K^- \nu\bar{\nu}$ rate. We therefore consider a different ratio, which is defined by

$$R_{256} \equiv \frac{\int_0^{s_m} ds\, dB(B^- \to K^- \nu\bar{\nu})/ds}{\int_0^{0.25} ds\, dB(B^- \to K^- l^+ l^-)/ds + \int_{0.6}^{s_m} ds\, dB(B^- \to K^- l^+ l^-)/ds} \tag{52}$$

In this ratio the fully integrated rate of $B^- \to K^- \nu\bar{\nu}$ is divided by the integrated rate of $B^- \to K^- l^+ l^-$ with only the narrow-resonance region removed. This ensures use of the maximal statistics in both channels. Due to the missing region in $B^- \to K^- l^+ l^-$ the dependence on the form factor shape will no longer be eliminated completely, but we still expect a reduced dependence. Numerically we obtain, using the same input as before,

$$R_{256} = 14.60\,{}^{+0.28}_{-0.38}\,(a_0)\,{}^{+0.10}_{-0.02}\,(b_1)\,{}^{-0.80}_{+0.62}\,(\mu) \tag{53}$$

This estimate shows that the uncertainty from a_0 and b_1 is indeed very small, at a level of about $\pm 3\%$. With better empirical information on the shape of the spectrum this could be further improved.

We conclude that ratios such as those in (50) and (52), or similar quantities with modified cuts, are theoretically very well under control. They are therefore ideally suited for testing the standard model with high precision.

4.3 Precision observables: $\bar{B} \to \bar{K} l^+ l^-$ with lattice input

Until now our strategy has been to achieve accurate predictions by eliminating the form factor dependence altogether. A variant of our analysis consists in taking a single hadronic parameter, the form factor at one particular value of q^2, $f_+(s_0)$, $s_0 = q_0^2/m_B^2$, as additional theory input. The shape of the form factor can be fitted to the experimental spectrum as discussed in sec. 3.2. At the expense of one extra hadronic parameter it is then possible to probe short distance physics based on $\bar{B} \to \bar{K} l^+ l^-$ alone. The necessary input $f_+(s_0)$ could come from lattice QCD calculations.

This approach is analogous to the method pursued in [42] to determine $|V_{ub}|$ from $B \to \pi l \nu$. In this case lattice results on the $B \to \pi$ form factor at a typical value of $q^2 = 16\,\mathrm{GeV}^2$ were considered as theory input. Experimental data on the spectrum and decay rate of $B \to \pi l \nu$ can then be used to extract $|V_{ub}|$. In order to describe the form factor shape, [42] employed dispersive bounds and a related class of general

parametrizations [43,44,45]. These more sophisticated parametrizations may also be applied in our case, if more than two shape parameters should be required to fit the data with the appropriate precision. For the time being the form factor parametrization used here is completely sufficient. As shown in [42], the limiting factor is the value of $f_+(s_0)$. We remark that in the case of $\bar{B} \to \bar{K} l^+ l^-$, the narrow-resonance region should be removed from the analysis when performing the fit to the form factor shape.

To illustrate the method we extract the form factor $f_+(s_0)$ at point $s_0 = 16\,\mathrm{GeV}^2/m_B^2$ from the measurement in (3). Using the best-fit shape parameters $a_0 = 1.6$ and $b_1/b_0 = 1$ we obtain

$$f_+(s_0) = 1.05 \pm 0.06\,(\mathrm{BR}) \pm 0.03\,(\mu) \tag{54}$$

where the first error is from the measured branching ratio in (3) and the second error is from scale dependence. The sensitivity to the shape parameters is comparable to the uncertainty from the branching ratio.

Unlike the parameters a_0 and b_1, the value of $f_+(s_0)$ determined in this way, assuming the standard model, is sensitive to the normalization of the branching ratio and therefore to new physics effects. These could be detected through a comparison with QCD calculations of $f_+(s_0)$. Our default choice of hadronic parameters leads to $f_+(s_0) = 1.16$ but the uncertainty is larger than 15%. Within the coming five to ten years a precision of $\pm 4\%$ might be achieved for the form factor $f_+(s_0)$ in lattice QCD [26].

5 New physics

The branching fractions of $B \to K\nu\bar{\nu}$ and $B \to K l^+ l^-$ are sensitive to physics beyond the standard model. If the new physics would modify both of them by (almost) the same factor, this change would not be visible in the ratios R_{25} or R_{256} discussed in sec. 4. In that case, the new physics could still be seen by studying $B \to K l^+ l^-$ separately with the method described in sec. 4.3. An example is a scenario with modified Z-penguin contributions [5] interfering constructively with the standard model terms. In this case R_{25} is changed only by a small amount.

In general, however, nonstandard dynamics will have a different impact on $B \to K\nu\bar{\nu}$ and $B \to K l^+ l^-$. The excellent theoretical control over the ratios R_{25} or R_{256} will help to reveal even moderate deviations from standard model expectations.

One example is the scenario with modified Z-penguin contributions [5] mentioned before, if these contributions interfere destructively with those of the standard model. In that case the ratios R_{25} or R_{256} could be significantly suppressed. The modified Z-penguin scenario may be realized, for instance, in supersymmetric models [5,46].

Another class of theories that do change the ratios are those where $B \to K l^+ l^-$ remains standard model like while $B \to K\nu\bar{\nu}$ receives an enhancement (or a suppression). Substantial enhancements of $B(B \to K\nu\bar{\nu})$ are still allowed by experiment, in fact much more than for $B \to K l^+ l^-$.

A first example are scenarios with light invisible scalars S contributing to $B \to KSS$ [46]. This channel adds to $B \to K\nu\bar{\nu}$, which is measured as $B \to K + \text{invisible}$. If the

scalars have nonzero mass, $B \to KSS$ could be distinguished from $B \to K\nu\bar{\nu}$ through the missing-mass spectrum. On the other hand, if the mass of S is small, or the resolution of the spectrum is not good enough, a discrimination of the channels may be difficult. The corresponding increase in $B(B \to K\nu\bar{\nu})$ could be cleanly identified through the ratios R_{25} and R_{256}.

A second example is given by topcolor assisted technicolor [47]. A typical scenario involves new strong dynamics, together with extra Z' bosons, which distinguishes the third generation from the remaining two. The resulting flavour-changing neutral currents at tree level may then predominantly lead to transitions between third-generation fermions such as $b \to s\nu_\tau\bar{\nu}_\tau$. An enhancement of $B(B \to K\nu\bar{\nu})$ would result and might in principle saturate the experimental bound (1). An enhancement of 20%, which should still be detectable, would probe a Z'-boson mass of typically $M_{Z'} \approx 3\,\mathrm{TeV}$. A similar pattern of enhanced $B \to K\nu\bar{\nu}$ and SM like $B \to Kl^+l^-$ is also possible in generic Z' models [46].

A more detailed exploration of new physics in $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$ is beyond the scope of this article. The examples mentioned above illustrate that the ratios of branching fractions considered here exhibit a significant sensitivity to interesting new physics scenarios. The subject of new physics in $b \to s\nu\bar{\nu}$ transitions has been discussed in [48] and most recently in [46]. New physics in $B \to Kl^+l^-$ has been studied in [49], including the information from angular distributions.

6 Conclusions

In this paper we have studied precision tests of the standard model through a combined analysis of $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$. The main points can be summarized as follows:

- After removing the narrow-resonance region the branching fraction of $B \to Kl^+l^-$ can be reliably computed. The dominant amplitude from semileptonic operators is simply a calculable expression times the form factor f_+. QCD factorization for low q^2 and OPE for high q^2 allow one to treat also matrix elements of 4-quark operators in a systematic way. These are dominated by charm loops, which are numerically small contributions to begin with. Since the tensor form factor f_T can be related to f_+ in the heavy-quark limit, the entire $B \to Kl^+l^-$ amplitude becomes calculable in terms of practically a single hadronic quantity, the form factor $f_+(s)$.

- The decay mode $B \to K\nu\bar{\nu}$ is a particularly clean process. It is completely determined by short-distance physics at the weak scale up to the same form factor $f_+(s)$. For the charged mode the background due to $B^- \to \tau^-\bar{\nu}_\tau \to K^-\nu_\tau\bar{\nu}_\tau$ should be subtracted from the experimental signal, but this will be possible without introducing any appreciable uncertainty.

- The form factor uncertainty can be eliminated by constructing suitable ratios of (partially) integrated rates such as R_{25} in (50) and R_{256} in (52). The resulting

quantities can be computed with high accuracy. The cancellation of form factors is exact and does not require the use of approximate flavour symmetries.

- The perturbative uncertainty of the ratios is estimated to be $\pm 5\%$ at next-to-leading order (NLO). This can be further improved by a NNLO analysis along the lines of [28]. Uncertainties from other sources are at the level of several percent. Some refinements in controlling them should still be possible.
- Based on the current measurements of $B^- \to K^- l^+ l^-$ we predict

$$B(B^- \to K^- \nu\bar{\nu}) = (3.64 \pm 0.47) \cdot 10^{-6}\,, \tag{55}$$

  at present the most accurate estimate of this quantity.
- The ratios of $B \to K\nu\bar{\nu}$ and $B \to Kl^+l^-$ rates have an interesting sensitivity to new physics. The new physics reach benefits from the high accuracy of the standard model predictions. A complementary new physics test is possible based on $B \to Kl^+l^-$ alone, exploiting experimental information on the q^2 spectrum, if f_+ at one particular value of q^2 is used as input from lattice QCD.
- New physics in the Wilson coefficients factorizes from low-energy hadronic physics in a simple way. The latter is essentially contained only in f_+. Our analysis can thus be generalized to specific new physics scenarios in a straightforward manner.

Our proposal puts $B \to K\nu\bar{\nu}$ as a new physics probe in the same class as $K \to \pi\nu\bar{\nu}$, the 'golden modes' of kaon physics. $B \to K\nu\bar{\nu}$ together with $B \to Kl^+l^-$ thus hold exciting opportunities for B physics at a Super Flavour Factory.

A Relation between form factors f_T and f_+

The three form factors in (13) and (14), $f_T(s)$, $f_+(s)$, and $f_0(s)$ or equivalently

$$f_-(s) \equiv [f_0(s) - f_+(s)]\, \frac{m_B^2 - m_K^2}{q^2} \tag{56}$$

are related in the heavy-quark limit. If we multiply (13) and (14) by $v_\mu \equiv p_\mu/m_B$ and use the equation of motion for the heavy quark, $\not{v}b = b$, we find

$$\frac{2m_B}{m_B + m_K} f_T = f_+ - f_- \tag{57}$$

Similarly, multiplying (13) with v_μ, using $\not{v}b = b$, and comparing the result with $q_\mu \cdot$ (13), where the quark equations of motion are used on the left-hand side, one finds

$$f_+ = -f_- \tag{58}$$

Together (57) and (58) imply

$$\frac{f_T(s)}{f_+(s)} = \frac{m_B + m_K}{m_B} \tag{59}$$

The relations (57) – (59) have been obtained in [50] in the heavy-quark limit. They apply immediately to the case where the kaon is soft, since then the heavy-quark mass is the only large energy scale in the problem. On the other hand, the derivation makes no explicit reference to the kaon energy and it has been argued in [51] that these relations should be valid in the entire kinematic domain. This conjecture can be justified within the framework of soft-collinear effective theory (SCET) [52,53], using the form factor relations in the large recoil limit [16,17], which also lead to (57) – (59). A related discussion can be found in [54].

We may thus use (59) in the entire range of q^2 between 0 and $(m_B - m_K)^2$. Since this expression for f_T/f_+ is an asymptotic result in the heavy-quark limit $m_b \gg \Lambda_{\rm QCD}$, an important issue is the question of subleading terms. These can be power corrections in $\Lambda_{\rm QCD}/m_B$ and perturbative QCD corrections. The perturbative corrections were computed in the heavy-quark and large recoil energy limit in [17] with the result (in the NDR scheme with $\overline{\rm MS}$ subtraction)

$$\begin{aligned}\frac{m_B}{m_B + m_K}\frac{f_T(s)}{f_+(s)} = 1 &- \frac{\alpha_s(\mu) C_F}{4\pi}\left[\ln\frac{\mu^2}{m_b^2} + \frac{4E_K}{m_B - 2E_K}\ln\frac{2E_K}{m_B}\right] \\ &- \frac{\alpha_s(\mu_h) C_F}{4\pi}\frac{4\pi^2 f_B f_K}{N f_+(s) E_K \lambda_B}\int_0^1 du\, \frac{\phi_K(u)}{1-u}\end{aligned} \tag{60}$$

where $\mu = \mathcal{O}(m_b)$, $\mu_h = \mathcal{O}(\sqrt{\Lambda m_b})$, and $E_K = \mathcal{O}(m_b)$ depends on $s = q^2/m_B^2$ through (5). We note that this relation remains valid when the kaon is soft, with $E_K = \mathcal{O}(\Lambda)$. In that case expression (60) simplifies. The term with $\alpha_s(\mu_h)$ is no longer perturbative since the effective scale μ_h becomes soft. However, the entire term is power suppressed $\sim \Lambda/m_b$ because $f_B \sim 1/\sqrt{m_b}$ and $f_+(1) \sim \sqrt{m_b}$. The second term in square brackets is also power suppressed and only the first correction $\sim \alpha_s \ln(\mu/m_b)$ survives. The α_s corrections can be consistently taken into account at NNLO even though at present, for low q^2, the second term from hard spectator interactions still introduces an uncertainty of about 5 – 10%.

Power corrections to the heavy-quark limit are more difficult to compute. An estimate can be obtained from light-cone QCD sum rules [23], which indicate that f_T/f_+ deviates from $1 + m_K/m_B$ by less than ±5% for $0 < s < 0.5$. The sum rule calculations include α_s corrections within their framework.

From these considerations we conclude that the relation (59) should be correct to within ±10%.

B Weak annihilation in $\bar{B} \to \bar{K} l^+ l^-$

Weak annihilation contributes to $\bar{B}^0 \to \bar{K}^0 l^+ l^-$ through QCD penguin operators. These induce the transition $b\bar{d} \to s\bar{d}$ where the valence quarks $b\bar{d}$ of the $\bar{B}^0$ meson are anni-

hilated and transformed into the constituents of the final state $\bar{K}^0$. The virtual photon producing the lepton pair may be emitted from any of the four quarks in this transition. In a similar manner the process $b\bar{u} \to s\bar{u}$ gives rise to weak annihilation in $B^- \to K^- l^+ l^-$, where the transition comes from QCD penguins and from doubly Cabibbo suppressed tree operators.

To be specific we treat the case of $\bar{B}^0 \to \bar{K}^0 l^+ l^-$ first. Here the leading-power contribution to weak annihilation comes from the QCD penguin operators

$$\begin{aligned} Q_3 &= (\bar{d}_i b_j)_{V-A} (\bar{s}_j d_i)_{V-A} + \ldots \\ Q_4 &= (\bar{d} b)_{V-A} (\bar{s} d)_{V-A} + \ldots \end{aligned} \tag{61}$$

The ellipsis refers to similar terms with d replaced by u, c, s and b, which do not contribute at the order we are considering. Colour indices are denoted by i, j.

The kinematics of the annihilation process is conveniently described in terms of two lightlike four-vectors $n_\pm$. Their components can be chosen, without loss of generality, as

$$n_\pm^\mu = (1, 0, 0, \pm 1) \tag{62}$$

The momenta of the B meson, the kaon and the lepton pair, p, k and q, respectively, can then be written as

$$p = \frac{m_B}{2}(n_+ + n_-), \qquad k = \frac{m_B^2 - q^2}{2m_B} n_+, \qquad q = \frac{q^2}{2m_B} n_+ + \frac{m_B}{2} n_- \tag{63}$$

For now we assume that the dilepton mass q^2 counts as order Λm_b, appropriate for the low-q^2 region. This means that momentum q is nearly lightlike and approximately in the direction of n_-, whereas k is lightlike and has the direction of n_+ if we neglect the kaon mass.

Consider next the $b\bar{d} \to s\bar{d}$ annihilation diagram where the virtual photon is emitted from the $\bar{d}$ in the initial state. We will denote by A_{d1} the contribution of this diagram to the $\bar{B}^0 \to \bar{K}^0 l^+ l^-$ matrix element of Q_4. Viewed as a function of the $\bar{d}$ four-momentum $l = \mathcal{O}(\Lambda)$, the diagram has the form

$$F(l) = F^{(0)}(l_+) + l_\perp^\mu \, F_\mu^{(1)}(l_+) \tag{64}$$

up to terms with a relative power suppression in Λ/m_b. The momentum l is decomposed into light-cone coordinates $l_\pm = n_\mp \cdot l$ and $l_\perp$, $n_\pm \cdot l_\perp = 0$, with respect to the two vectors $n_\pm$ in (62). The expression for the light-cone projector of the B meson in momentum space, appropriate for an amplitude of the type shown in (64), has been derived in [17]. It is given by

$$b\bar{d} \equiv i \frac{f_B m_B}{4} \frac{1+\not{v}}{2} \left[\phi_+(\omega) \, \not{n}_+ + \phi_-(\omega) \left(\not{n}_- - \omega \gamma_\perp^\nu \frac{\partial}{\partial l_\perp^\nu} \right) \right] \gamma_5 \tag{65}$$

The derivative in (65) extracts the $F^{(1)}$ contribution in (64). After it has been applied, $l_\perp$ has to be set to zero and l_+ is identified with ω.

Calculating the contribution A_{d1} with the projector in (65) we find

$$A_{d1} = e^2 f_B f_K \, (\bar{u}\not{k}v) \left[\frac{2Q_{d1}}{m_B} \int_0^\infty d\omega \, \frac{\phi_-(\omega)}{\omega - q^2/m_B} + \frac{Q_{d1}}{q^2} \right] \tag{66}$$

Here $\bar{u}\gamma_\mu v$ is the lepton current in momentum space and $Q_{d1} = -1/3$ the down-quark charge. The second term in (66) is of the same order in Λ/m_b as the first term. It has a pole in q^2 and is inconsistent with electromagnetic gauge invariance. However, it is structure independent (it depends only on f_B, f_K, not on the distribution amplitudes) and is cancelled by corresponding contributions from the remaining three diagrams. These diagrams, where the photon is emitted from the b quark, the s quark and the final-state $\bar{d}$ quark ($d2$) give explicitly

$$A_b + A_s + A_{d2} = -e^2 f_B f_K \, (\bar{u}\not{k}v) \frac{Q_b - Q_s + Q_{d2}}{q^2} \tag{67}$$

as contributions to the matrix element of Q_4. Charge conservation implies that $Q_b - Q_s + Q_{d2} \equiv Q_{d1}$, which guarantees the cancellation of the $1/q^2$ term in (66) by (67). The first term in (66) then leads to the result (29) once the matrix element of Q_3 is included, which is the same as the one of Q_4 up to a factor 1/3 from colour.

We remark that the $1/q^2$ term and half of the first term in (66) come from the $F^{(1)}$ part in (64). The latter contribution, as well as all $1/q^2$ terms are absent at leading power for weak annihilation in $B \to K^*\gamma$ [28,55], in contrast to the present case.

The result in (29) develops a logarithmic singularity when q^2 becomes soft of order Λ^2, corresponding to the formal limit $q^2 \to 0$. Since the singularity is integrable, the soft region is power suppressed in comparison to the branching ratio integrated from 0 to $q^2 \sim \Lambda m_b$, as has been discussed in [28].

The annihilation contribution in (32) from operators $Q_5 = -2(\bar{d}_i b_j)_{S-P}(\bar{s}_j d_i)_{S+P} + \ldots$ and $Q_6 = -2(\bar{d}b)_{S-P}(\bar{s}d)_{S+P} + \ldots$ is obtained in analogy to the case of Q_3, Q_4. Also for Q_5, Q_6 there are structure-independent terms $\sim 1/q^2$, which cancel when all four diagrams are added. The remaining result is again due to the diagram where the photon is emitted from the $\bar{d}$ quark in the initial state.

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References

[1] M. Bona *et al.*, arXiv:0709.0451 [hep-ex].

[2] T. Kageyama, AIP Conf. Proc. **842** (2006) 1064.

[3] T. Browder *et al.*, JHEP **0802** (2008) 110 [arXiv:0710.3799 [hep-ph]].

[4] T. E. Browder, T. Gershon, D. Pirjol, A. Soni and J. Zupan, arXiv:0802.3201 [hep-ph].

[5] G. Buchalla, G. Hiller and G. Isidori, Phys. Rev. D **63** (2001) 014015 [arXiv:hep-ph/0006136].

[6] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.

[7] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex]; http://www.slac.stanford.edu/xorg/hfag/

[8] K. F. Chen *et al.* [BELLE Collaboration], Phys. Rev. Lett. **99** (2007) 221802 [arXiv:0707.0138 [hep-ex]].

[9] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **94** (2005) 101801 [arXiv:hep-ex/0411061].

[10] J. T. Wei *et al.* [BELLE Collaboration], arXiv:0904.0770 [hep-ex].

[11] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **73** (2006) 092001 [arXiv:hep-ex/0604007].

[12] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68** (1996) 1125 [arXiv:hep-ph/9512380].

[13] G. Buchalla and A. J. Buras, Nucl. Phys. B **400** (1993) 225; M. Misiak and J. Urban, Phys. Lett. B **451** (1999) 161 [arXiv:hep-ph/9901278]; G. Buchalla and A. J. Buras, Nucl. Phys. B **548** (1999) 309 [arXiv:hep-ph/9901288].

[14] A. J. Buras and M. Münz, Phys. Rev. D **52** (1995) 186 [arXiv:hep-ph/9501281].

[15] M. Misiak, Nucl. Phys. B **393** (1993) 23 [Erratum-ibid. B **439** (1995) 461].

[16] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **60** (1999) 014001 [arXiv:hep-ph/9812358].

[17] M. Beneke and T. Feldmann, Nucl. Phys. B **592** (2001) 3 [arXiv:hep-ph/0008255].

[18] M. B. Wise, Phys. Rev. D **45** (1992) 2188.

[19] G. Burdman and J. F. Donoghue, Phys. Lett. B **280** (1992) 287.

[20] A. F. Falk and B. Grinstein, Nucl. Phys. B **416** (1994) 771 [arXiv:hep-ph/9306310].

[21] R. Casalbuoni *et al.*, Phys. Rept. **281** (1997) 145 [arXiv:hep-ph/9605342].

[22] G. Buchalla and G. Isidori, Nucl. Phys. B **525** (1998) 333 [arXiv:hep-ph/9801456].

[23] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232].

[24] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D **70** (2004) 094002 [arXiv:hep-ph/0407226].

[25] P. Ball and R. Zwicky, Phys. Lett. B **633** (2006) 289 [arXiv:hep-ph/0510338].

[26] M. Artuso *et al.*, Eur. Phys. J. C **57** (2008) 309 [arXiv:0801.1833 [hep-ph]].

[27] D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478** (2000) 417 [arXiv:hep-ph/9904490].

[28] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B **612** (2001) 25 [arXiv:hep-ph/0106067].

[29] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606** (2001) 245 [arXiv:hep-ph/0104110].

[30] A. G. Grozin and M. Neubert, Phys. Rev. D **55** (1997) 272 [arXiv:hep-ph/9607366].

[31] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Eur. Phys. J. C **61** (2009) 439 [arXiv:0902.4446 [hep-ph]].

[32] M. Beneke, eConf **C0610161** (2006) 030 [Nucl. Phys. Proc. Suppl. **170** (2007) 57] [arXiv:hep-ph/0612353].

[33] S. W. Bosch and G. Buchalla, JHEP **0501** (2005) 035 [arXiv:hep-ph/0408231].

[34] M. B. Voloshin, Phys. Lett. B **397** (1997) 275 [arXiv:hep-ph/9612483].

[35] A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, Phys. Lett. B **402** (1997) 167 [arXiv:hep-ph/9702318].

[36] G. Buchalla, G. Isidori and S. J. Rey, Nucl. Phys. B **511** (1998) 594 [arXiv:hep-ph/9705253].

[37] B. Grinstein and D. Pirjol, Phys. Rev. D **70** (2004) 114005 [arXiv:hep-ph/0404250].

[38] F. Krüger and L. M. Sehgal, Phys. Lett. B **380** (1996) 199 [arXiv:hep-ph/9603237].

[39] J. F. Kamenik and C. Smith, arXiv:0908.1174 [hep-ph].

[40] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **77** (2008) 011107 [arXiv:0708.2260 [hep-ex]].

[41] I. Adachi *et al.* [Belle Collaboration], arXiv:0809.3834 [hep-ex].

[42] T. Becher and R. J. Hill, Phys. Lett. B **633** (2006) 61 [arXiv:hep-ph/0509090].

[43] C. G. Boyd and M. J. Savage, Phys. Rev. D **56** (1997) 303 [arXiv:hep-ph/9702300].

[44] C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. Lett. **74** (1995) 4603 [arXiv:hep-ph/9412324].

[45] C. Bourrely, B. Machet and E. de Rafael, Nucl. Phys. B **189** (1981) 157.

[46] W. Altmannshofer, A. J. Buras, D. M. Straub and M. Wick, JHEP **0904** (2009) 022 [arXiv:0902.0160 [hep-ph]].

[47] G. Buchalla, G. Burdman, C. T. Hill and D. Kominis, Phys. Rev. D **53** (1996) 5185 [arXiv:hep-ph/9510376].

[48] Y. Grossman, Z. Ligeti and E. Nardi, Nucl. Phys. B **465** (1996) 369 [Erratum-ibid. B **480** (1996) 753] [arXiv:hep-ph/9510378].

[49] C. Bobeth, G. Hiller and G. Piranishvili, JHEP **0712** (2007) 040 [arXiv:0709.4174 [hep-ph]].

[50] N. Isgur and M. B. Wise, Phys. Rev. D **42** (1990) 2388.

[51] N. Isgur, Phys. Rev. D **43** (1991) 810.

[52] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63** (2001) 114020 [arXiv:hep-ph/0011336].

[53] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65** (2002) 054022 [arXiv:hep-ph/0109045].

[54] R. J. Hill, Phys. Rev. D **73** (2006) 014012 [arXiv:hep-ph/0505129].

[55] S. W. Bosch and G. Buchalla, Nucl. Phys. B **621** (2002) 459 [arXiv:hep-ph/0106081].